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Inflation in Brans–Dicke Theory for the Radiation Universe

Marcelo Samuel Berman[1] and M. M. Som[2]

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The Brans–Dicke (BD) theory admits de Sitter spacetime as a solution with an equation of state $p=\frac{1}{3}\rho$ when the coupling constant ω of the BD theory is $-3/2$.

1. INTRODUCTION

It is now generally accepted that inflation is the embryonic phase in the evolution of the observable universe. In standard classical cosmology there is no model for the phase transitions associated with the inflation. One drives the inflation by a symmetry-breaking phase transition, based upon the dynamical evolution of a very weakly coupled scalar field displaced from the minimum of its potential.

As the cosmological symmetry-breaking phase transition is not essential for the inflation, one can associate growing instabilities of a cosmological model with the phase transitions. In this context the Brans–Dicke theory is a better candidate for the de Sitter spacetime, as it is unstable (Berman, 1989*a*). There exist some interesting vacuum solutions (O'Hanlon and Tupper, 1972) in the de Sitter spacetime that can be related to the inflationary phase. If one assumes that in the early universe, the scalar field in the BD theory is the dominant field, i.e., the Newtonian constant G is negligibly small, the dynamics of the scalar field might be associated with the exponential expansion of the universe.

[1]Department of Physics and Astronomy, University of Alabama, Tuscaloosa, Alabama 35487-0324.
[2]Departamento de Fisica Matematica do Instituto de Fisica da Universidade Federal do Rio de Janeiro, Rio de Janeiro, RJ, 21945, Brazil.

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McIntosh (1973) gave two particular time-dependent solutions for $\omega = 0$ and $\omega = -\frac{1}{2}$, where ω is BD coupling constant. However, his solutions do not correspond to a perfect fluid with known equation of state. Later Berman and Som (1989) and Berman (1989*b*) obtained solutions with de Sitter spacetime, but their solutions are not valid for radiation or incoherent dust.

In the present paper, we consider the dynamics of the scalar field of the BD theory when it is comparable to the radiation field ($p = \frac{1}{3}\rho$). It is found that the BD theory admits an exponential solution of the de Sitter type when the coupling constant assumes the special value $\omega = -3/2$. Such a solution might be relevant in the very early universe when the matter content is ultrarelativistic and the equation of state can be approximated by that of the radiation field.

2. BD FIELD EQUATIONS

The field equations of the BD theory are given by

$$G^i_j = 8\pi\phi^{-1}T^i_j + \omega\phi^{-2}(\phi^i\phi_j - \tfrac{1}{2}\delta^i_j\phi^k\phi_k) + \phi^{-1}(\phi^i_{;j} - \delta^i_j\phi^k_{;k}) \tag{1}$$

where

$$T^\mu_{M\nu} = (\rho + p)U^\mu U_\nu + \delta^\mu_\nu p \tag{2}$$

and

$$\Box^2\phi = g^{\mu\nu}\phi_{;\mu;\nu} \tag{3}$$

The energy-momentum tensor T^μ_ν obeys the usual conservation law

$$T^\mu_{\nu;\mu} = 0 \tag{4}$$

The scalar field ϕ is assumed to obey a field equation

$$(3 + 2\omega)\,\Box^2\phi = T^\mu_{M\mu} \tag{5}$$

We consider $\phi = \phi(r, t)$ in the homogeneous and isotropic Robertson–Walker spacetime given by

$$ds^2 = -dt^2 - R^2(t)(dr^2 + r^2\,d\theta^2 + r^2\sin^2\theta\,d\phi^2) \tag{6}$$

A straightforward calculation, using (2), gives the time–time component of equation (1) as

$$\begin{aligned} 3(\dot{R}/R)^2 = \rho/\phi + (\omega/2\phi^2)[\dot{\phi}^2 + (\phi')^2/R^2] - 1/\phi \\ \times[3(\dot{R}/R)\dot{\phi} - 1/R^2(2\phi'/r + \phi'')] \end{aligned} \tag{7}$$

while the space–space component of equation (1) gives

$$2\ddot{R}/R+(\dot{R}/R)^2=-P/\phi-(\omega/2\phi^2)[\dot{\phi}^2+(\phi')^2/R^2]-1/\phi \times[\ddot{\phi}+2\dot{R}/R\dot{\phi}-2\phi'/rR^2] \tag{8}$$

$$2\ddot{R}/R+(\dot{R}/R)^2=-p/\phi-(\omega/2\phi^2)[\dot{\phi}^2-(\phi')^2/R^2]-1/\phi \times[\ddot{\phi}+2\dot{R}/R\dot{\phi}-1/R^2(\phi'/r+\phi'')] \tag{9}$$

The spacetime component of equation (1) gives

$$\omega\dot{\phi}\phi'+\phi(\dot{\phi}'-\dot{R}/R\phi')=0 \tag{10}$$

where

$$\dot{\phi}=\frac{\partial\phi}{\partial t} \quad \text{and} \quad \phi'=\frac{\partial\phi}{\partial t}$$

The field equation for ϕ reads

$$(3+2\omega)[(\phi''+2\phi'/r)/R^2-(\dot{\phi}'+3\dot{R}/R\dot{\phi})]=3p-\rho \tag{11}$$

and the conservation law (4) gives

$$\dot{\rho}=-3\dot{R}/R(\rho+p) \tag{12}$$

When $\phi\neq 0$, equation (10) can be written as

$$\omega\dot{\phi}/\phi+\dot{\phi}'/\phi'=\dot{R}/R \tag{13}$$

Subtracting (8) from (9) yields

$$\phi''/p+\omega\phi'/\phi=1/r \tag{14}$$

From equations (13) and (14) one obtains the general solution (O'Hanlon and Tupper, 1972)

$$\phi=\{(\omega+1)[Ar^2R(t)+B(t)]\}^{1/(\omega+1)} \tag{15}$$

where A is an arbitrary constant and $B(t)$ is an arbitrary function of (t). It is interesting to note that (15) was obtained earlier by O'Hanlon and Tupper (1972) in the context of vacuum-field solutions, but, as we have shown, it is valid in the general case.

3. RADIATION FIELD

For the radiation field one obtains

$$(3+2\omega)\,\Box^2\phi=0 \tag{16}$$

If $\omega \neq -3/2$ one obtains vacuum solutions (O'Hanlon and Tupper, 1972) in de Sitter spacetime for $\omega = -4/3$. If $\omega = -3/2$, the constant A and the function $B(t)$ of (15) are to be determined from equations (7)–(9). Equation (12) rules out (15) as a solution for $R/R = H$, where H is a constant.

We consider now $\phi = \phi(t)$ and take

$$\phi = A\rho \tag{16'}$$

Since $\phi' = 0$, equation (10) is identically satisfied. From (12) one obtains, using (16′) and $p = \frac{1}{3}\rho$,

$$\dot{\phi}/\phi = -4(\dot{R}/R) \tag{17}$$

Substituting (17) in (7) and (8) gives, with $-\omega = -3/2$,

$$3(\dot{R}/R)^2 = A \tag{18}$$

$$2\ddot{R}/R + (\dot{R}/R)^2 = -1/3A + 4(\dot{R}/R)^2 \tag{19}$$

Combining (18) and (19), one obtains

$$\ddot{R}/R = (\dot{R}/R)^2 = H \tag{20}$$

Equation (20) is the well-known de Sitter spacetime.

4. CONCLUSION

We have shown that in the BD theory the homogeneous scalar field ϕ is the inflation field for the coupling constant $\omega = -3/2$ even in the presence of the radiation field. In the very early universe, the matter content is approximated as an ideal gas composed of ultrarelativistic particles. An equation of state identical to that of the radiation field ($p = \frac{1}{3}\rho$) is an acceptable equation of state for not too high a density. For such a gas, the de Sitter spacetime is a natural solution of the BD field equations.

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